# Auxiliary field QMC for fermions



## Outline

1) Odd and even leg Hubbard models.

Predefined lattices: N_leg_lattice

ARTICLE

## Surprises on the Way from One- to Two-Dimensional Quantum Magnets: The Ladder Materials

Elbio Dagotto and T. M. Rice

Science 271 (1996), no. 5249, 618–623.

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**ARTICLE**

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2) Emergent SO(4) symmetry in the one-dimensional Hubbard model.

$$\langle \hat{\boldsymbol{S}}_i \hat{\boldsymbol{S}}_{i+r} \rangle \propto (-1)^r / r$$

$$\langle \hat{K}_i \hat{K}_{i+r} \rangle - \langle \hat{K}_i \rangle \langle \hat{K}_{i+r} \rangle \propto (-1)^r / r, \quad \hat{K}_i = \sum_\sigma \left( \hat{c}^\dagger_{i,\sigma} \hat{c}_{i+1,\sigma} + H.c. \right)$$

Up to logarithmic corrections:

$$\langle \hat{D}_i \hat{D}_{i+r} \rangle - \langle \hat{D}_i \rangle \langle \hat{D}_{i+r} \rangle \propto (-1)^r / r, \quad \hat{D}_i = \hat{\boldsymbol{S}}_i \hat{\boldsymbol{S}}_{i+1}$$

Dimer-Dimer correlations are in predefined observables.

Note: Field theory is O(4) non-linear sigma model in 1+1 dimensions with WZW term.

$$S\left[\hat{\varphi}\right] = \int dx d\tau \frac{1}{G} \left( \partial_\mu \hat{\varphi}(x,\tau) \right)^2 + 2\pi i \Gamma\left[\hat{\varphi}\right], \quad \Gamma\left[\hat{\varphi}\right] = \frac{1}{\mathrm{Area}\left(S^3\right)} \int_0^1 du \int dx d\tau \epsilon_{\alpha,\beta,\gamma,\delta} \hat{\varphi}_\alpha \partial_x \hat{\varphi}_\beta \partial_\tau \hat{\varphi}_\gamma \partial_u \hat{\varphi}_\delta$$

ALF simulations of O(5) non-linear sigma model in 2+1 d with WZW term → Z. Wang et al. Phys. Rev. Lett. 126 (2021), 045701

For an explicit calculation see supplemental material of
T. Sato, M. Hohenadler, T, Grover, J. McGreevy, and F. F. Assaad, Topological terms on topological defects: a quantum Monte Carlo study, arXiv:2005.08996 (2020).

## Projects



3) a) SU(N) Hubbard model on the one-dimensional chain. Show that the ground state at N = 4 is dimerized.

b) Can you write a program for the SU(N) quantum antiferromagnetic in the self-adjoint antisymmetric representation? (See ALF 2.0 documentation, Section on the SU(N) Kondo lattice. )

3) a) SU(N) Hubbard model on the one-dimensional chain. Show that the ground state at N = 4 is dimerized.

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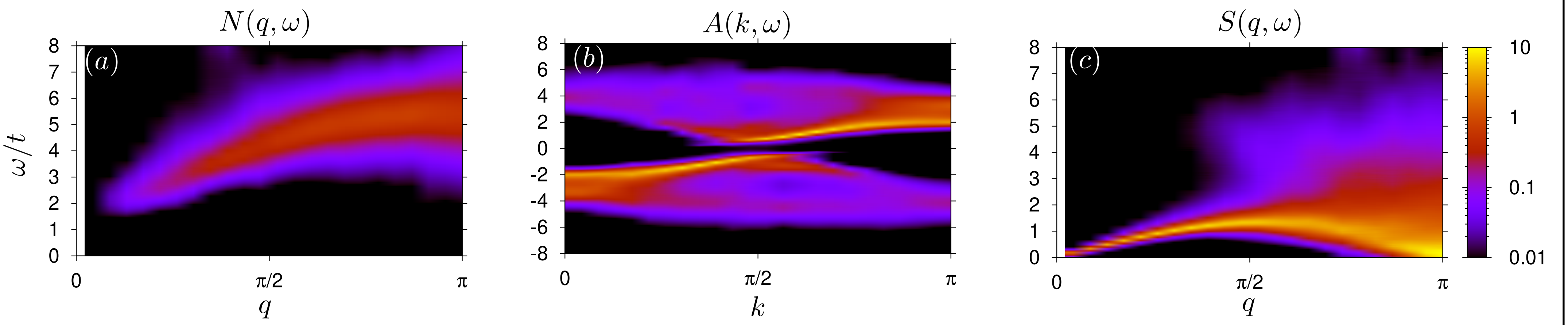
4) Dynamics of one-dimensional Hubbard chains. Understand how to use Maxent ( see Documentation Chapter 10) to produce:

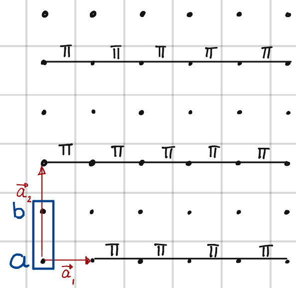
$$\langle n \rangle = 1, U/t = 4, \beta t = 10, L = 46$$



5) Correlation effects in Chern bands.

No sign problem for negative U, and arbitrary filling.

For positive U, no sign problem only at half-filling ( $\mu = 0$ )




$$H = H_{Dirac} + H_{QSH} + U \sum_i \Big[ \sum_\sigma \big( a^\dagger_{i\sigma} a_{i\sigma} - \tfrac{1}{2} \big) \Big]^2 + U \sum_i \Big[ \sum_\sigma \big( b^\dagger_{i\sigma} b_{i\sigma} - \tfrac{1}{2} \big) \Big]^2 - \mu \sum_{i,\sigma} \big( a^\dagger_{i\sigma} a_{i\sigma} + b^\dagger_{i\sigma} b_{i\sigma} \big)$$

$$H_{Dirac} = -t \sum_{i,\sigma} \Big[ a^\dagger_{i,\sigma} \big( -a_{i+a_1,\sigma} + b_{i,\sigma} \big) + b^\dagger_{i,\sigma} \big( a_{i+a_2,\sigma} + b_{i+a_1,\sigma} \big) + h.c. \Big]$$

$$H_{QSH} = \lambda \sum_{i\sigma} i\sigma \big( a^\dagger_{i,\sigma} b_{i+a_1,\sigma} + b^\dagger_{i,\sigma} a_{i+a_1,\sigma} - b^\dagger_{i,\sigma} a_{i+a_1+a_2,\sigma} - a^\dagger_{i,\sigma} b_{i-a_2+a_1,\sigma} \big) + h.c.$$

PHYSICAL REVIEW B **102**, 201112(R) (2020)

Rapid Communications Editors' Suggestion

**Superconductivity, pseudogap, and phase separation in topological flat bands**

Johannes S. Hofmann,[1] Erez Berg,[1,*] and Debanjan Chowdhury[2,†]
[1]*Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel*
[2]*Department of Physics, Cornell University, Ithaca, New York 14853, USA*

# Projects



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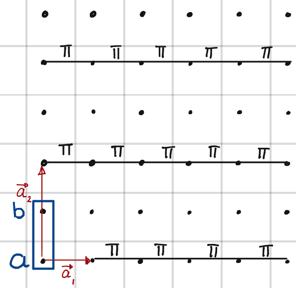


$$H = H_{Dirac} + H_{QSH} + U \sum_i \Big[ \sum_\sigma \big( a^\dagger_{i\sigma} a_{i\sigma} - \tfrac{1}{2} \big) \Big]^2 + U \sum_i \Big[ \sum_\sigma \big( b^\dagger_{i\sigma} b_{i\sigma} - \tfrac{1}{2} \big) \Big]^2 - \mu \sum_{i,\sigma} \big( a^\dagger_{i\sigma} a_{i\sigma} + b^\dagger_{i\sigma} b_{i\sigma} \big)$$

$$H_{Dirac} = -t \sum_{i,\sigma} \Big[ a^\dagger_{i,\sigma} \big( -a_{i+a_1,\sigma} + b_{i,\sigma} \big) + b^\dagger_{i,\sigma} \big( a_{i+a_2,\sigma} + b_{i+a_1,\sigma} \big) + h.c. \Big]$$

$$H_{QSH} = \lambda \sum_{i\sigma} i\sigma \big( a^\dagger_{i,\sigma} b_{i+a_1,\sigma} + b^\dagger_{i,\sigma} a_{i+a_1,\sigma} - b^\dagger_{i,\sigma} a_{i+a_1+a_2,\sigma} - a^\dagger_{i,\sigma} b_{i-a_2+a_1,\sigma} \big) + h.c.$$

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6) Edge physics.
Consider the above model with open boundary conditions in the y-direction. Along the edge you should be able to investigate the physics of a helical Luttinger liquid.

PRL **106**, 100403 (2011) PHYSICAL REVIEW LETTERS week ending 11 MARCH 2011

**Correlation Effects in Quantum Spin-Hall Insulators: A Quantum Monte Carlo Study**

M. Hohenadler, T. C. Lang, and F. F. Assaad

7) Sticking issues. Consider the doped attractive Hubbard model. Both HS decompositions based on

$$H_U = -\frac{U}{2}\left(n_{\boldsymbol{i},\uparrow} - n_{\boldsymbol{i},\downarrow}\right)^2$$

Mz = True in Hubbard Hamiltonian.

$$H_U = \frac{U}{2}\left(n_{\boldsymbol{i},\uparrow} + n_{\boldsymbol{i},\downarrow} - 1\right)^2$$

Mz = False in Hubbard Hamiltonian.

are free of the negative sign problem. Check autocorrelation times (see Documentation Sec. 4) for the particle number as a function of doping for *large* values of |U| and assess which choice the HS transformation is more efficient.

8) Investigate the spinless t-V model on a $\pi-$flux lattice. As a function of V, you should observe a transition in the Gross-Neveu Ising universality class to a charge density wave state.

PHYSICAL REVIEW D **101,** 074501 (2020)

**Fermion-bag inspired Hamiltonian lattice field theory for fermionic quantum criticality**

Emilie Huffman[1] and Shailesh Chandrasekharan[2]

9) Consider the half-filled Kondo lattice model on the Honeycomb lattice. Show that there is direct magnetic order disorder transition as a function of J. The transition is a consequence of the competition between the RKKY and Kondo interactions.

PHYSICAL REVIEW B, VOLUME 63, 155114

**Spin and charge dynamics of the ferromagnetic and antiferromagnetic two-dimensional half-filled Kondo lattice model**

S. Capponi and F. F. Assaad

10) Dzyaloshinskii-Moriya

$$\hat{H} = \sum_{\langle i,j\rangle} \left( J \, \hat{\vec{S}}_i \cdot \hat{\vec{S}}_j + \vec{D}_{i-j} \cdot \hat{\vec{S}}_i \times \hat{\vec{S}}_j \right) = \sum_{\langle i,j\rangle} \left[ J \, \hat{\vec{S}}_i \hat{\vec{S}}_j + \left| \varepsilon_{\alpha\beta\gamma} D^{\alpha}_{i-j} \right| \left( \hat{S}^{\beta}_i + \mathrm{sign}\left(\varepsilon_{\alpha\beta\gamma} D^{\alpha}_{i-j}\right) \hat{S}^{\gamma}_j \right)^2 \right]$$

Fermionize. $\hat{\vec{S}}_i = \sum_{s,s'} \hat{f}^{\dagger}_{i,s} \frac{\vec{\sigma}_{s,s'}}{2} \hat{f}_{i,s'}$ and impose the constraint, $\sum_s \hat{f}^{\dagger}_{i,s} \hat{f}_{i,s} = 1$, by including a Hubbard U:

PHYSICAL REVIEW B **104**, L081106 (2021)

Letter

**Quantum Monte Carlo simulation of generalized Kitaev models**

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11) Define your own problem